Design of Adaptive Structures under Random Impact Conditions

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Summary. The methodology (based on the so-called Dynamic Virtual Distortion Method) of the design of structures exposed to impact loading is presented in the work. Minimization of material volume and accelerations of structural response are chosen as the objective functions for optimal design of structures adapting to impact loads. The cross-sections of structural members as well as stress levels triggering plastic-like behavior and initial prestressing are the design parameters. A general formulation of this problem, as well as particular cases, are discussed.

Key words: adaptive structures, optimal control, dynamic sensitivity analysis

1 Introduction – Problem Formulation

Motivation for the undertaken research is to respond to requirements for high impact energy absorption e.g. in the structures exposed to the risk of extreme blast, light, thin wall tanks with high impact protection, vehicles with high crashworthiness, protective barriers, etc. Typically, the suggested solutions focus on the design of passive energy absorbing systems. These systems are frequently based on the aluminum and/or steel honeycomb packages characterized by a high ratio of specific energy absorption. However high is the energy absorption capacity of such elements, they still remain highly redundant structural members, which do not carry any load in the actual operation of a given structure. In addition, passive energy absorbers are designed to work effectively in pre-defined impact scenarios. For example, the frontal honeycomb cushions are very effective during a symmetric axial crash of colliding cars, but are completely useless in other types of crash loading. Consequently, distinct and sometimes completely independent systems must be developed for specific collision scenarios.

In contrast to the standard passive systems, the proposed approach focuses on active adaptation of energy absorbing structures (equipped with a sensor

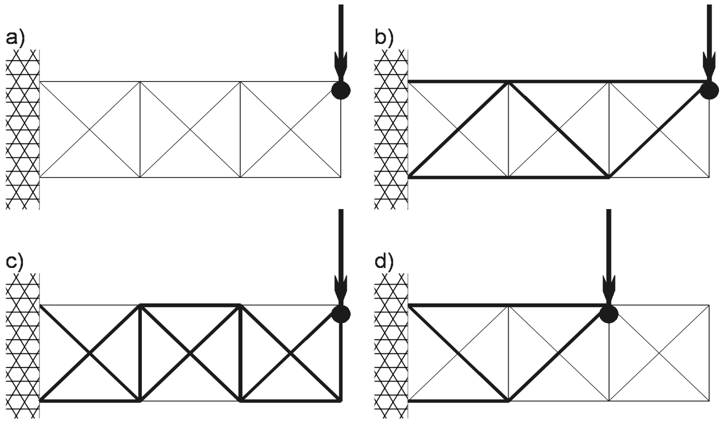


Fig. 1. Testing example of truss-like cantilever. a) the initial configuration, b) the stiffest substructure, c) the most compliable substructure, d) the stiffest substructure designed for the second load case VDM Based Dynamic Analysis of Adaptive Structure

system detecting impact in advance and controllable semi-active dissipaters, so called structural fuses) with a high ability of adaptation to extreme overloading. The concept formulation and first numerical analysis are based on the previously published paper [4]. Various formulations of crashworthiness-based structural design problem are presented in papers [1, 2, 8, 9, 10, 11–14], while the adaptive crashworthiness concept has been first proposed in [3] and [6]. The optimal design methodology proposed below combines sensitivity analysis with the redesign process, allowing optimal redistribution of material as well as stress limit control in structural fuses. It is assumed that this 'smart' devices are able to release structural connections in a controlled way, triggering plastic-like distortions mimicking elasto-plastic behaviour shown in Fig. 2.

The objectives in design of impact absorbers are the following:

- to sustain all expected impact loads and especially the one with maximal impact energy,
- to absorb all expected impact loads in the most smooth way, minimizing maximal accelerations.

Assuming first only one state of impact load, with possible variation of impact intensity, let us take into account the following definitions:

E^u – maximal expected impact energy
σ^u – yield stress level for ideal elasto-plastic material used to built the structure
β^u – maximal allowed plastic-like distortion to be generated in structural fuses.

Now, assume that in hiperstatic truss-like structure (k redundant) k independent states of self-equilibrated stresses can be generated in such a way that the yield stress level $\sigma^\star \leq \sigma^u$ is reached (under a given load) at the same time in all structural members. Then, the minimal volume of material V necessary to dissipate the maximal expected impact energy E^u can be calculated from the following formula: $E^u = V\sigma^u\beta^u$, requiring uniformly distributed maximal stresses triggering plastic distortions in all structural members. Note, that some initial state of virtual distortions $\beta^{0\prime}$ has to be introduced in order to generate desired prestress described above.

Having defined the minimum volume of structural material let us discuss the heuristic methodology for structural remodeling on the example of truss-like cantilever shown in Fig. 1a. and loaded dynamically in structural node (mass m hits the tip joint with the initial velocity v). The well-known solution for the optimal topology of the stiffest substructure S_1 (made from the determined material volume) is shown in Fig. 1b (cf. [7]) while the most compliant substructure S_2 is demonstrated in Fig. 1c. Both of these substructures are isostatic and the material distribution (sizing of members' cross-sections) guarantees uniformly distributed locally extremal stresses in response to the impact $I(m,v)$.

Let us now calibrate the substructures S_1 and S_2 defined above in such way, that the volumes of the material for both of them are identical and equal to $V/2$. Finally, let us compose the resultant structure $(S_1 \cup S_2)$, where the cross-section of each element i is defined as the following mean value: $\mu_i = (\mu_i^1 + \mu_i^2)/2$, and where the coefficient μ_i^1 ($\mu_i^1 = A_i/A_i'$, A_i and A_i' denote modified and the initial cross-section of the member i, respectively) describes the ratio of cross-section modification for the element i from the substructure S_1, while a coefficient μ_i^2 describes the ratio of cross-section modification for the element i from the substructure S_2. In the example shown in Fig. 1 the topology of the resultant complex structure is the same as the topology of the initial configuration (Fig. 1a).

The **remodeling** process described above is related to the following **first problem** formulation:

$$\min V = \min \sum_{\mathrm{i}} \mu_i A_i' l_i \tag{1}$$

subject to the following constraints:

$$\begin{gathered} \left|\beta_i^0(t)\right| \leq \beta^u \\ |\sigma_i(t)| \leq \sigma^\star \leq \sigma^u \\ \beta_i^0(t)\sigma_i(t) \geq 0 \end{gathered} \tag{2}$$

where l_i denotes the length of the member i and stresses $\sigma_i(t)$ depend on the maximal expected impact load $I(m,v)$ and the control parameters: μ_i, $\sigma_i^\star$, $\beta_i^{0\prime}$, what will be discussed in the next section. The constraint (2) describes the condition of dissipative character of plastic-like distortions generation.

Having determined the material remodeling described above, the following, **second problem of adaptation** to the identified (in advance, using a sensor system) impact load can be formulated:

$$\min \max \ddot{u}_i(t) \tag{3}$$

subject to the following constraints:

$$|u_i(t)| \leq u^u \tag{4}$$

where displacements $u_i(t)$ depend on the identified in real time impact load $I(m,v)$, previously determined material redistribution μ_i and the control parameters: $\sigma_i^\star$, $\beta_2^{0\prime}$ describing plastic-like adaptation and the initial prestress, what will be discussed in the next section.

The following strategies of adaptation to the identified in real time impact loads have to be considered:

- detachment of elements i: $(i \in S_2) \wedge (i \notin S_1)$ and tuning proper stress levels $\sigma^\star$ allows to obtain the most compliable substructure S_2, able to receive (the most smoothly) small impacts with constraint (4) still satisfied,
- detachment of elements i: $(i \in S_1) \wedge (i \notin S_2)$ and tuning proper stress levels $\sigma^\star$ allows to obtain the stiffest substructure S_1, able to receive stronger impacts without excessive deflections violating constraint (4),
- introducing proper initial distortions $\beta_2^{0\prime}$ in elements of the substructure S_1, and tuning stress levels $\sigma^\star$ allows to obtain the uniform distribution of the stress level $\sigma^\star$ in all structural elements and simultaneous triggering plastic-like distortions, what leads to the most effective impact energy dissipation. This third strategy should be applied when the impact capacity of the second one is too low.

Note, that *detachment* of structural elements mentioned in the two first strategies can be realized introducing properly determined initial distortions $\beta_2^{0\prime}$.

In multi-load cases compromise solutions for the material redistribution have to be taken into account. For example, the same impact $I(m,v)$ provoked by different masses and initial velocities leads to different mass distribution (μ_i). Optimal solutions for slow impacts (bigger mass and lower initial velocities) require mass transportation to the support neighborhood while in the case of fast impact (the same impact energy provoked by smaller mass hitting the structure with higher velocity) the mass transportation to the neighborhood of loaded node can be observed (cf. [7]). Therefore, in case of expected variable m/v ratio, the optimal material redistribution requires a compromise solution. Considering optional impact load state (cf. Fig. 1d), the topology of the stiffest substructure is different from the previous one (Fig. 1b), what also demonstrates that the determinations of the compromise solution for a multi-load remodeling problem require new numerical tools able to tackle this challenging problem. The proposition of new numerical technique (based on so called Virtual Distortion Method, VDM, cf. [5]) able to solve the problem discussed above will be discussed in the next two sections.

2 VDM Based Dynamic Analysis of Adaptive Structure

In this chapter we will formulate the VDM based description of the dynamic response of elasto-plastic truss structure. Applying discretized time description, the evolution of strains and stresses (with respect to initial cross-sections) can be expressed as follows:

$$\varepsilon_i(t) = \varepsilon_i^L(t) + \sum_{\tau \le t} \sum_j D_{ij}(t-\tau) \cdot \varepsilon_j^0(\tau) + \sum_{\tau \le t} \sum_k D_{ik}(t-\tau) \cdot \beta_k^0(\tau) \tag{5}$$

$$\begin{aligned} \sigma_i'(t) =& E_i \left(\varepsilon_i(t) - \varepsilon_i^0(t) - \beta_i^0(t) \right) \\ \sigma_i'(t) =& E_i \Bigg[\varepsilon_i^L(t) + \sum_{\tau \le t} \sum_j D_{ij}(t-\tau) \cdot \varepsilon_j^0(\tau) - \varepsilon_i^0(t) + \\ & \sum_{\tau \le t} \sum_k D_{ik}(t-\tau) \cdot \beta_k^0(\tau) - \beta_i^0(t) \Bigg] \end{aligned} \tag{6}$$

where so called dynamic influence matrices $D_{ij}(t-\tau)$ describe the strain evolution caused in the truss element member i and in time instance t, due to unit virtual distortion impulse generated in member j in the time instant τ. The vector $\varepsilon_i^L(t)$ denotes the strain evolution due to external loads applied to the elastic structure with initial material distribution (unmodified cross-sections of members), $\varepsilon_i^0(t)$ denotes virtual distortions responsible for modification of design variables and $\beta_i^0(t)$ describes plastic-like distortions. Note that the matrix D stores information about the properties of the entire structure (including boundary conditions) and describes dynamic (not static) structural response to a locally generated impulse of virtual distortion. Note also that the influence of local modifications of design variables on the stiffness matrix only, was assumed in further analysis. The full analysis taking into account the

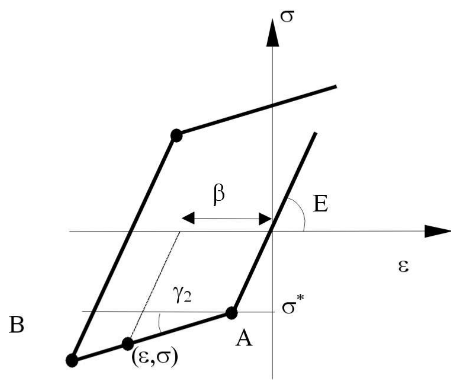


Fig. 2. Piece-wise linear constitutive relation for the adaptive structural member

influence of virtual distortions $\varepsilon_i^0(t)$ on both, the stiffness as well as the mass matrices is more complicated and will be discussed in separate publication. From now on, we assume that small Latin index j runs through all modified members, and small Latin index k runs through all plastified elements.

In order to take into account elasto-plastic structural behaviour, let us use the bilinear constitutive model with hardening (Fig. 2), given by the equation (7)

$$\sigma_i(t) - \sigma_i^\star = \gamma_i E_i \left(\varepsilon_i(t) - \varepsilon_i^\star\right) \tag{7}$$

where $\sigma_i^\star$ denotes plastic yield stress, γ_i denotes hardening parameter and E_i denotes Young's modulus.

Now, when we substitute stress (6) and strain (5) evolution in time to the formula (7) we obtain the following equation:

$$\begin{aligned}\beta_i^0(t) = (1-\gamma_i)\left(\varepsilon_i^L(t) - \varepsilon_i^\star\right) + (1-\gamma_i)\sum_{\tau\le t}\sum_j D_{ij}^D\,(t-\tau)\cdot\varepsilon_j^0\,(\tau)\\ + (1-\gamma_i)\sum_{\tau\le t}\sum_k D_{ik}^H\,(t-\tau)\cdot\beta_k^0\,(\tau)\end{aligned} \tag{8}$$

Taking advantage of two expressions for the internal forces applied to the so called distorted (9) (with modification of material distribution modeled through virtual distortions) and modified (10) (with redesigned cross-sections from A to A', without imposing virtual distortions) structure:

$$P_i(t) = E_i A_i \left(\varepsilon_i(t) - \varepsilon_i^0(t) - \beta_i^0(t)\right) \tag{9}$$

$$P_i(t) = E_i A_i' \left(\varepsilon_i(t) - \beta_i^0(t)\right) \tag{10}$$

A formula combining components $\varepsilon_i^0(t)$ and $\beta_i^0(t)$ can be derived, where these components are non zero only for distorted and/or plastified elements.

If we assume that forces and strains in both structures: *distorted* (9) and *modified* (10) are the same, the modifications simulated with virtual distortion can be combined with these distortions through the flowing formula:

$$\varepsilon_i^0(t) = (1-\mu_i)\left(\varepsilon_i(t) - \beta_i^0(t)\right) \tag{11}$$

where $\varepsilon_i(t)$ describes strain in member i in time t, while $\mu_i = A_i/A_i'$ denotes ratio of the new cross-section to the initial one. Parameter $\mu_i \in \langle 0, 1\rangle$ specifies the size of modification of cross-sections in element i. If $\mu_i = 1$ that means that in element i the cross-section does not change, and if $\mu_i = 0$ that means that element i can be neglected in the analysis.

The formula (11) can be rewritten in the following form (12):

$$\mu_i = \frac{A_i}{A_i'} = \frac{\varepsilon_i(t) - \varepsilon_i^0(t) - \beta_i^0(t)}{\varepsilon_i(t) - \beta_i^0(t)} \tag{12}$$

Now let us substitute strain evolution in time (5) to formula (12) getting the following set of equations:

$$\varepsilon_i^0(t) = (1-\mu_i)\,\varepsilon_i^L(t) + \sum_{\tau\le t}\sum_j D_{ij}^D(t-\tau)\cdot\varepsilon_j^0(\tau) + \sum_{\tau\le t}\sum_k D_{ik}^H(t-\tau)\cdot\beta_k^0(\tau) - \beta_i^0(t) \quad (13)$$

Note that the equations (8) are not dependant on the virtual distortions responsible for modification of design variables in time t, but only on the distortions in previous time steps $\varepsilon(\tau)$ because of the assumption (14).

$$D_{ij}(0) = 0 \quad (14)$$

Therefore, the plastic-like distortions $\beta_i^0(t)$ should be calculated first in each time step of the algorithm.

Equations (8) and (13) need only computation of the right-hand side expressions, and we need not solve the coupled sets of equations.

Formulas (8) and (13) allow us to compute the virtual distortions' development in time, modeling both: assumed remodeling of material distribution as well as adapted plastic-like stress limits.

If there is no plasticity in our problem, then plastic-like distortions are equal to zero and the equation (13) takes the following form:

$$\varepsilon_i^0(t) = (1-\mu_i)\,\varepsilon_i^L(t) + \sum_{\tau\le t}\sum_j D_{ij}(t-\tau)\cdot\varepsilon_j^0(\tau). \quad (15)$$

Analogously, if there is no remodeling, distortions are equal to zero (the parameter μ_i is equal to one) and equation (8), determining plastic like distortions' development takes the following form:

$$\beta_i^0(t) = (1-\gamma_i)\left(\varepsilon_i^L(t) - \varepsilon_i^\star\right) + (1-\gamma_i)\sum_{\tau\le t}\sum_k D_{ik}^H(t-\tau)\cdot\beta_k^0(\tau) \quad (16)$$

To prove that the VDM method gives the same solutions as commercial programs let us compare results for a simple truss structure shown in Fig. 3 with the structural response determined with ANSYS.

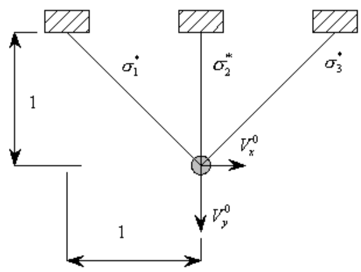


Fig. 3. Testing example truss structure. (Young modulus $E_i = 2.1 \times 10^11\,\mathrm{Pa}$, cross-sections $A_i = 1 \times 10^{-4}\,\mathrm{m}$, density $\rho_i = 7800\,\mathrm{kg/m^3}$)

All elements have different yield stress limits $\sigma_i^\star$ as well as parameters responsible for modification of design variables μ_i.

$$\sigma_1^\star = 8 \times 10^7\text{Pa}, \qquad \sigma_2^\star = 4 \times 10^7\text{Pa}, \qquad \sigma_3^\star = 6 \times 10^7\text{Pa}$$
$$\mu_2 = 0.7, \qquad \mu_1 = 0.5, \qquad \mu_3 = 0.9$$

In the lower node, concentrated mass 20 kg is added, together with the following initial condition (modeling with external object):

$$V_x^0 = 3\,\text{m/s}, \quad V_y^0 = 5\,\text{m/s}.$$

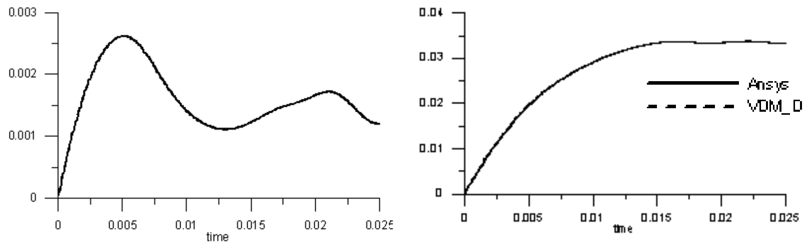


Fig. 4. Strain evolution in time for elements: a) left element, b) central element

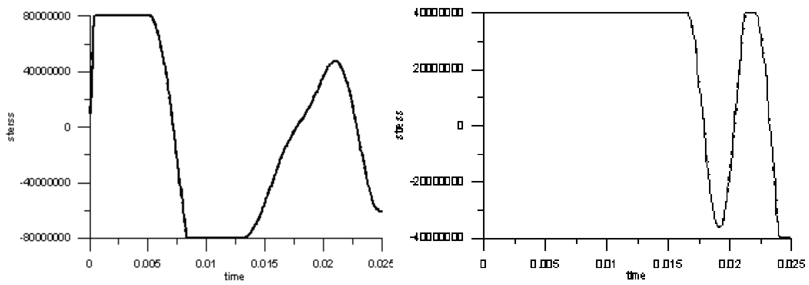


Fig. 5. Stress evolution in time for elements: a) left element, b) central element

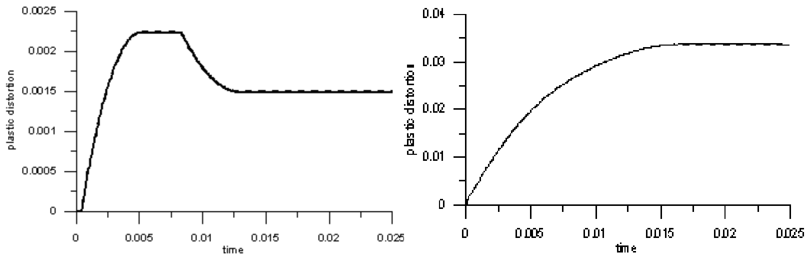


Fig. 6. Plastic distortion evolution in time for elements: a) left element, b) central element

On the graphs, the comparison of strain (Fig. 4), stress (Fig. 5) and plastic distortion (Fig. 6) development for the first and the second element, respectively is demonstrated.

Note, that only modification of the stiffness matrix (due to remodeling) has been taken into account in the above formulas. Analogous modification of the mass matrix has to be added in order to describe the complex remodeling phenomena. However, it was decided to keep this presentation simpler without additional complication of formulas.

3 Gradient Based Approach to Structural Remodeling

Let us demonstrate now, how the VDM based approach supports sensitivity analysis, useful in optimization procedures. Assume that the objective function is defined as maximization of dissipated energy during the adaptation process, given by the following formula:

$$U \max = \sum_t \sum_i \sigma_i(t) \Delta\beta_i(t) \mu_i A_i' l_i \tag{17}$$

subject to constrains:

$$-\tilde{\beta}_i \leq \beta_i \leq \tilde{\beta}_i, \quad 0 \leq \mu_i \leq 1, \quad \mu_i A_i' l_i = \text{const} \tag{18}$$

where $\tilde{\beta}_i$ denotes lower and upper limit imposed on plastic-like distortions, $\mu_i A_i' l_i$ denotes total volume of material which should be constant during the remodeling process. In this way, solving the above problem (17) for increasing load intensities, one can find the desired design of the structure with maximal impact load capacity, constraining considerations to small deformations.

To make further analysis more communicative, let us distinguish two particular cases. The first one, establishing the best material redistribution in all structural members leads to the determination of the following design variables: $\mu_i = A_i/A_i'$. In this case $\Delta\beta_k^0(t) = 0$, and $\Delta\beta_k^0(t)$ is replaced by $\varepsilon_i(t)$ in the objective function (17).

The gradient of this objective function can be calculated analytically and takes the following form:

$$\frac{\partial U}{\partial \mu_m} = \left(\frac{\partial U}{\partial \sigma_i(t)} \frac{\partial \sigma_i(t)}{\partial \varepsilon_j^0(t)} + \frac{\partial U}{\partial \varepsilon_i(t)} \frac{\partial \varepsilon_i(t)}{\partial \varepsilon_j^0(t)} \right) \frac{\partial \varepsilon_j^0(t)}{\partial \mu_m} + \frac{\partial U}{\partial \mu_m} \tag{19}$$

where the particular components can be expressed as follows:

$$
\begin{aligned}
\frac{\partial U}{\partial \sigma_i(t)} &= \sum_i \Delta\beta_i(t)\mu_i A'_i l_i \\
\frac{\partial U}{\partial \varepsilon_i(t)} &= \sum_i \sigma_i(t)\mu_i A'_i l_i \\
\frac{\partial U}{\partial \mu_m(t)} &= \sum_t \sigma_m(t)\varepsilon_m(t) A'_m l_m \\
\frac{\partial \sigma_i(t)}{\partial \varepsilon_j^0(t)} &= -\, E_i \delta_{ij} \\
\frac{\partial \varepsilon_j^0(t)}{\partial \mu_m} &= -\, \varepsilon_i^L(t) + \sum_{\tau \le t} \sum_j D_{ij}^D(t-\tau) \cdot \frac{\partial \varepsilon_j^0(\tau)}{\partial \mu_m}
\end{aligned}
\tag{20}
$$

In the second case we are looking for optimal distribution of yield stress limits in structural members and the design variables during optimization process are $\sigma_i^\star$. The corresponding gradient of the objective function, with respect to yield stress limits, takes the following form:

$$
\frac{\mathrm{d}U}{\mathrm{d}\sigma_l^\star} = \left(\frac{\partial U}{\partial \sigma_p(t)} \frac{\partial \sigma_p(t)}{\partial \Delta\beta_k^0(t)} + \frac{\partial U}{\partial \Delta\beta_k^0(t)} \right) \frac{\partial \Delta\beta_k^0(t)}{\partial \sigma_l^\star} \tag{21}
$$

where the new components of gradient can be expressed as follows:

$$
\frac{\partial \sigma_p(t)}{\partial \Delta\beta_k^0(t)} = -E_p \Delta\beta_p^0(t)\delta_{pk} \tag{22}
$$

$$
\frac{\partial \Delta\beta_k^0(t)}{\partial \sigma_l^\star} = -\frac{(1-\gamma_i)}{E} + (1-\gamma_i) \sum_{\tau \le t} \sum_k D_{ik}^H(t-\tau) \cdot \frac{\partial \beta_k^0(\tau)}{\partial \sigma_l^\star} \tag{23}
$$

Finally, the last case couples optimization sub-problems: *remodeling* end *adaptation* of adaptive structure. The design variables describe simultaneously material redistribution as well as yield stress limits: $\mu_i = A_i/A'_i$ and $\sigma_i^\star$, respectively.

The coupled gradient formula takes the following form:

$$
\begin{aligned}
\frac{\mathrm{d}U}{\mathrm{d}\sigma_l^\star} = \Bigg(\frac{\partial U}{\partial \sigma_p(t)} \bigg(\frac{\partial \sigma_p(t)}{\partial \varepsilon_j^0(t)} \frac{\partial \varepsilon_j^0(t)}{\partial \Delta\beta_k^0(t)} + \frac{\partial \sigma_p(t)}{\partial \Delta\beta_k^0(t)} \bigg) \\
+ \frac{\partial U}{\partial \Delta\beta_k^0(t)} + \frac{\partial U}{\partial \mu_m} \frac{\partial \mu_m}{\partial \Delta\beta_k^0(t)} \Bigg) \frac{\partial \Delta\beta_k^0(t)}{\partial \sigma_l^\star}
\end{aligned}
\tag{24}
$$

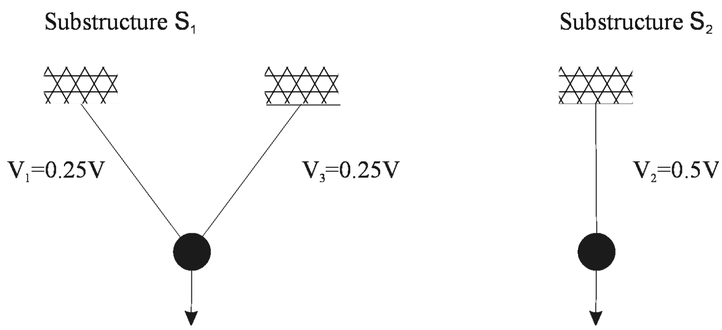
4 Design of Adaptive Structure (Small Deformations) – Testing Example

The coupled, gradient based remodeling problem described above is restricted to small deformations. However, it can be used as numerical tool for redesign of topology of adaptive structure on the base of analysis performed in the initial phase of structural response to the impact.

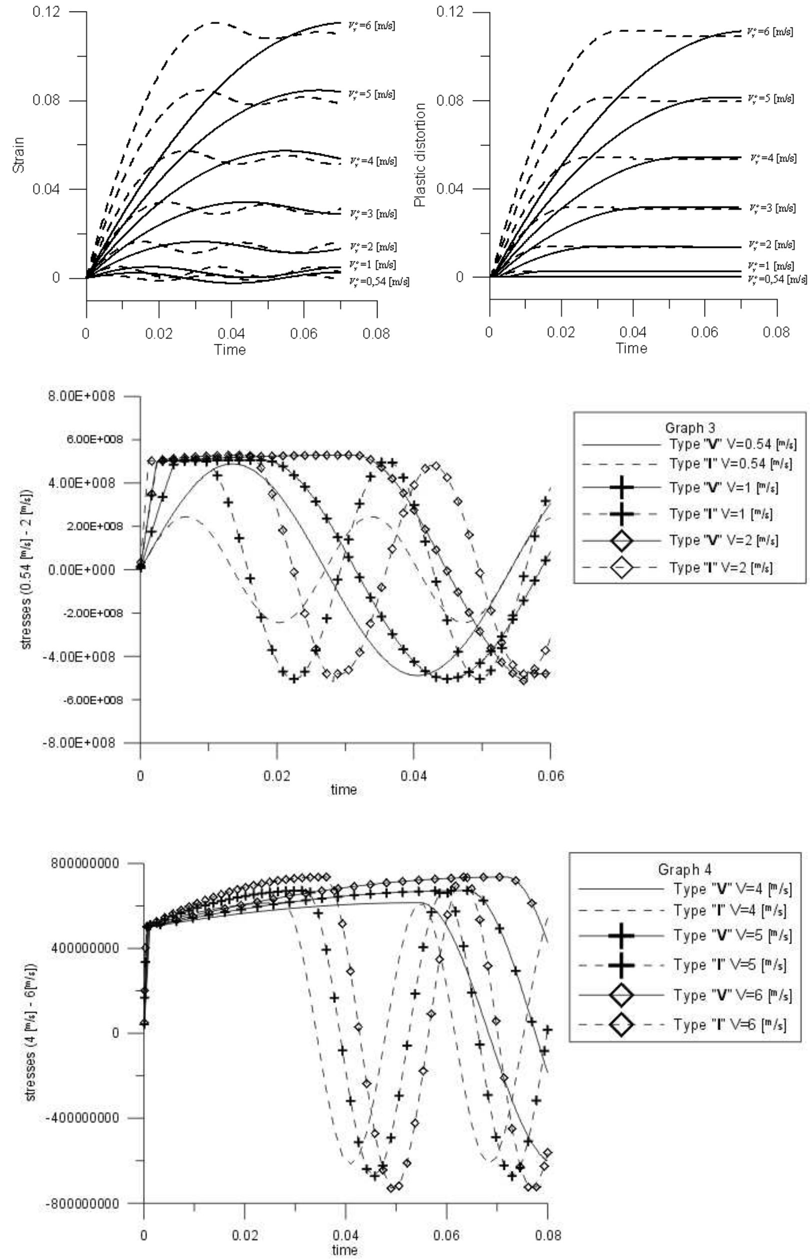
Let us demonstrate effectiveness of the proposed design concept of adaptive impact absorber on the base of simple truss structure shown in Fig. 3. The described above VDM technique (in development) will be soon applied to solve real design problems for the proposed smart impact absorbing type of structures.

Let us assume, that the testing structure of shock absorber (Fig. 3) is designed to absorb the impact of mass $m = 745\,\mathrm{kg}$ hitting (and sticking to the node afterwards) the node with its initial velocity v, where $v < 8.7\,\mathrm{m/s}$. It means that the maximal expected impact energy can reach $E^u = mv^2/2 =$ 28194.525 J. Consequently, the material volume used to built the structure should be calculated from the formula: $E^u = V\sigma^u\beta^u$ (cf. section 1) and is equal, assuming $\sigma^u = 5 \times 10^8\,\mathrm{Pa}$ and $\beta^u = 0.12$, to $V = 5.22 \times 10^{-5}\,\mathrm{m}^2$. Now, following the methodology proposed in section 1, let us decompose the structure into two substructures: S_1 and S_2 with the same volume of material: $V_1 = V_2 = V/2$. The structural dynamic responses for substructures S_1, S_2 and for the composed structure $S_1 \cup S_2$ are shown in Figs. 7b and 7c, respectively. Changing initial impact velocity in the range: $\langle 0.54\,\mathrm{m/s}, 8.7\,\mathrm{m/s}\rangle$ we can observe evolution of stresses, strains and plastic distortions in elements of the corresponding substructures S_1 and S_2 (Fig. 7b) and also the analogous evolutions in the compound structure $S_1 \cup S_2$ prestressed by introducing initial, plastic-like distortion $\beta_2^{0\prime} = 2.62 \times 10^{-3}$ into the vertical member of the structure (Fig. 7c).

a)



b)


Strain
Time
Plastic distortion
Time
Graph 3
Type "V" V=0.54 [m/s]
Type "I" V=0.54 [m/s]
Type "V" V=1 [m/s]
Type "I" V=1 [m/s]
Type "V" V=2 [m/s]
Type "I" V=2 [m/s]
stresses (0.54 [m/s] - 2 [m/s])
time
Graph 4
Type "V" V=4 [m/s]
Type "I" V=4 [m/s]
Type "V" V=5 [m/s]
Type "I" V=5 [m/s]
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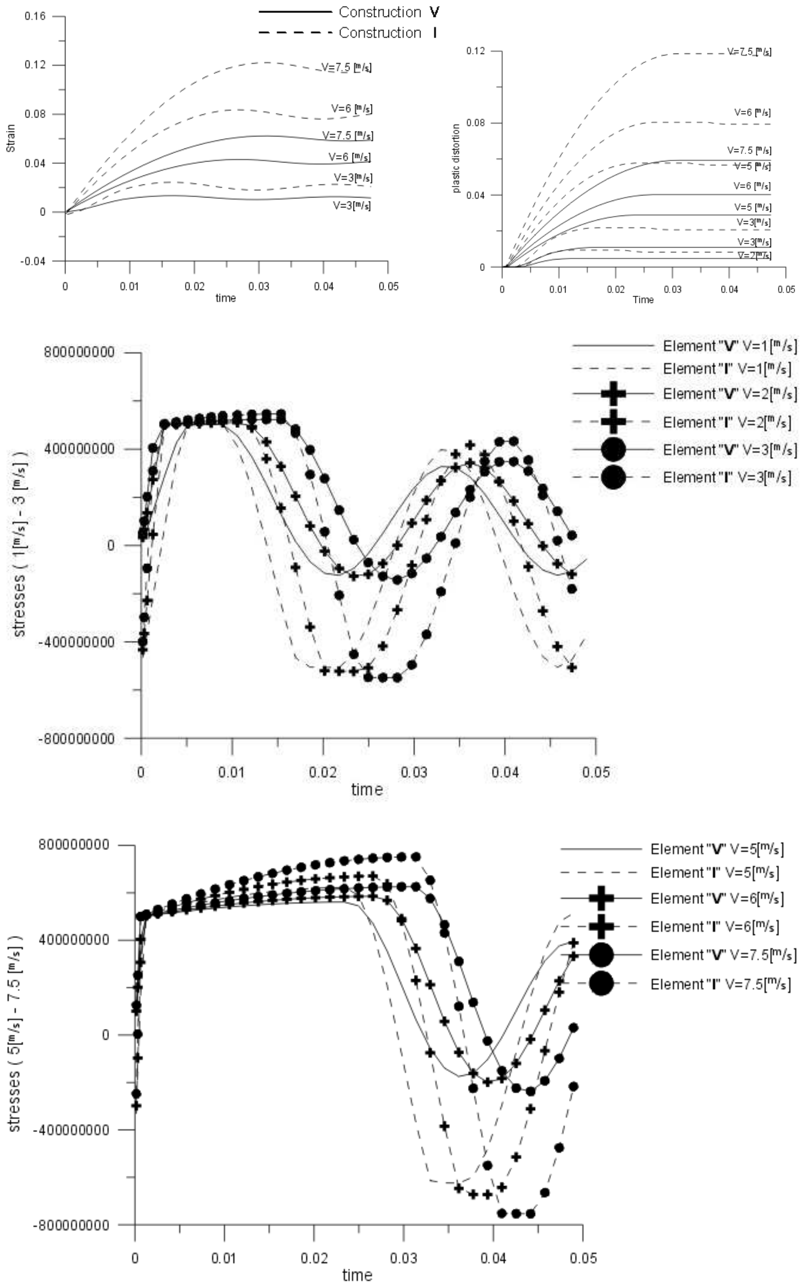
stresses (4 [m/s] - 6 [m/s])
time

c)


Construction V
Construction I
Strain
time
plastic distortion
Time
V=7.5 [m/s]
V=6 [m/s]
V=5 [m/s]
V=3 [m/s]
V=2 [m/s]
stresses (1[m/s] - 3 [m/s])
Element "V" V=1[m/s]
Element "I" V=1[m/s]
Element "V" V=2[m/s]
Element "I" V=2[m/s]
Element "V" V=3[m/s]
Element "I" V=3[m/s]
stresses (5[m/s] - 7.5 [m/s])
Element "V" V=5[m/s]
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Element "V" V=7.5[m/s]
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d)

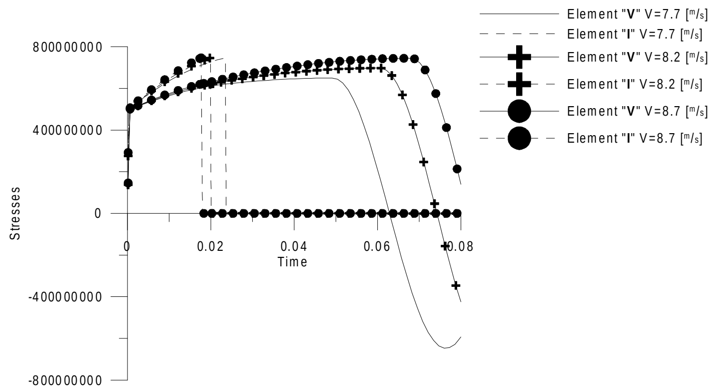

Fig. 7. Testing example of adaptive structure. a) decomposition into substructures S_1 and S_2, b) dynamic responses for substructures S_1 and S_2, c) dynamic response for the compound structure $S_1 \cup S_2$, d) stress evolution in time for the compound structure $S_1 \cup S_2$ with switching off the middle element

The following characteristic impact velocities can be identified: $v = 0.54\,\mathrm{m/s}$, when the substructure S_2 starts to yield, $v = 3.9\,\mathrm{m/s}$, when the deflection of the substructure S_2 reaches its assumed limitation $u^u = 0.108\,\mathrm{m}$, $v = 7.5\,\mathrm{m}$, when the plastic like distortion generated in substructure S_1 reaches its assumed limitation β^u, $v = 8.7\,\mathrm{m/s}$, when the plastic like distortion generated in compound structure $S_1 \cup S_2$ reaches its assumed limitation β^u. Then, analyzing numerical results presented in the Fig. 7, the following interpretation can be given. For the impact velocities $v < 0.54\,\mathrm{m/s}$ the adaptive structure responds elastically, without any energy dissipation, as the most compliant substructure S_2 is still too stiff. For the impact velocities $0.54\,\mathrm{m/s} < v < 3.9\,\mathrm{m/s}$ the most compliant substructure S_2 should be used for the impact absorption (substructure S_1 should be detached). For the impact velocities $3\,\mathrm{m/s} < v < 6.1\,\mathrm{m/s}$ the stiffest substructure S_1 should be used for the impact absorption (substructure S_2 should be detached). For the impact velocities $6\,\mathrm{m/s} < v < 8.7\,\mathrm{m/s}$ the prestressed compound structure $S_1 \cup S_2$ should be used for the impact absorption. Note, that the first mode of operation (S_2 substructure) allows to reach accelerations 280% smaller than in the second mode case (S_1 substructure) for small impacts. On the other hand, the third mode of operation ($S_1 \cup S_2$ structure) allows reaching impact capacity (in terms of energy dissipation) 110% higher than in the second mode case (S_1 substructure) for strong impacts. Note, that the limitation for impact absorption in the last case is due to active constraint β^u reached in the vertical member (cf.Fig. 7c). Then, switching off the connection between this member and the rest of the structure (cf.Fig. 7d) (semi-active adaptation in real time) allows extension of the structural impact capacity up to above 203% (comparing with S_1 substructure capacity) and the corresponding impact velocity, as the inclined members will still continue process of dissipation.

The crucial points in the proposed methodology are:

- development of technique for detection and identification of the impact in advance,
- development of devices for quickly responding, controlled in real time, semi-active detachment of structural members and for introducing initial distortions.

Another challenging field for research is mentioned above development of methodology able to determine optimal control strategies for impact absorption in multi-load cases, when compromise solutions are required.

5 Optimal Adaptation to Impact Load (Large Deformations)

Following the above described procedure for structural remodeling, let us assume that the structural geometry is already determined. Then, the second objective of the smoothest structural adaptation to identified impact can be addressed. Still constraining ourselves to small deformations, the optimal adaptation procedure based on the gradient calculation (7) can be proposed. However, the main contribution to the impact energy dissipation is due to the plastic flow, which makes an enlargement of strokes in controlled dissipaters crucial. In the consequence, large deformations have to be taken into account.

To provide optimal energy absorption it is necessary to perform a process of adaptation, consisting of the following three stages:

- Load identification
- Choosing optimal strategy
- Structural adaptation

The dynamic load level can be evaluated, in advance, before the impact by measuring velocity and estimating the mass of the colliding body. Alternatively , its value might be identified at the beginning of the impact process by sensors embedded into the structure.

In order to dissipate the kinetic energy in an optimal way, one has to apply a correct strategy to the active elements, where the following two strategies are formulated:

- semi-active control,
- active control.

In the first case pre-selected triggering stress levels σ_i in structural elements remain unchanged during impact, in the second one theoretical possibility of real-time changes in control parameters is assumed. Although full

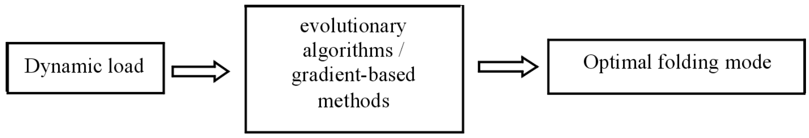

Fig. 8. The algorithm for semi-active control

real-time control does not seem to be feasible in a real design, one can expect that at least a few stress changes might be applied during the impact time.

The problem of optimal adaptation can be formulated as follows:

- Semi-active control – for a given impact load minimize the difference between acceleration values in selected points of the structure and desired response function $\ddot{q}_i^d(t)$:

$$\min f = \sum_t \sum_i \left[\ddot{q}_i(t) - \ddot{q}_i^d(t)\right]^2 \tag{25}$$

- Active control – for a given impact load, for every time step minimize the difference between acceleration values in selected points of the structure and desired response function $\ddot{q}_i^d(t)$:

$$\min f(t) = \sum_i \left[\ddot{q}_i(t) - \ddot{q}_i^d(t)\right]^2 \tag{26}$$

  subject to the following constraints

$$|\bar{\sigma}| \in \langle \bar{\sigma}_{\min}, \bar{\sigma}_{\max} \rangle\,, \quad \max\{q\} \leq \bar{q} \tag{27}$$

  where $\bar{\sigma}$ denotes the plastic-like yield stress level and $\bar{q}$ is the maximum crush distance.

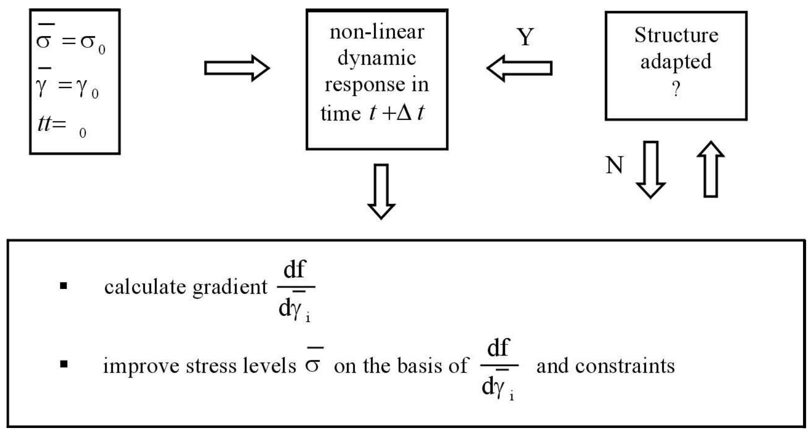

Fig. 9. The algorithm for active control

Satisfactory solution of the above problem exists if the external load intensity is not higher than the maximal safe load level. Beyond this limit a control strategy with the highest possible capability of energy dissipation must be applied.

The Multifolding Microstructure MFM

Let us discuss the truss-like microstructure (similar to honeycomb layout) shown on Fig. 10. Elements arranged in a periodic pattern are equipped with specially designed devices called micro-fuses. Micro-fuses provide control over yield stress in an element. After reaching selected threshold elements exhibit plastic-like behaviour.

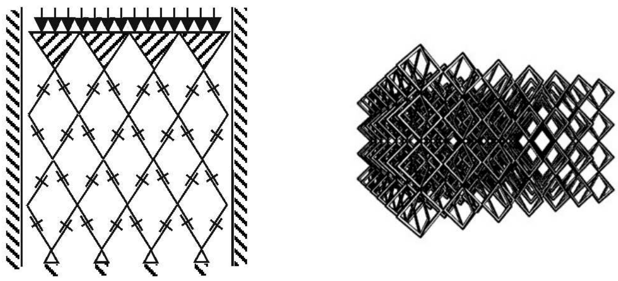

Fig. 10. Multifolding microstructure MFM

In order to get the additional value of energy dissipation (due to the synergy of repetitive use of dissipaters) the crucial point is to pre-design the optimal distribution of yield stress levels in all stickers, triggering desired sequence of local collapses.

Two single-column MFM models with a different number of control parameters are presented on Fig. 11. The most basic structure has only two control parameters (Fig. 11(b)): yield thresholds marked 1 and 2. Therefore, only two presented folding sequences are possible.

More complex structure “5” with five control thresholds is depicted in Fig. 11(a). The number of possible folding modes is in this case much greater that in the latter one.

It is clearly visible that different distribution of yield stresses will result in different behaviour of the MFM and different stiffness characteristics. Therefore, structure can adapt itself to the level of dynamic loading.

Because of complex phenomena like plasticity and contact, modeling of the MFM microstructure involves highly nonlinear and numerically expensive dynamic FEM analysis.

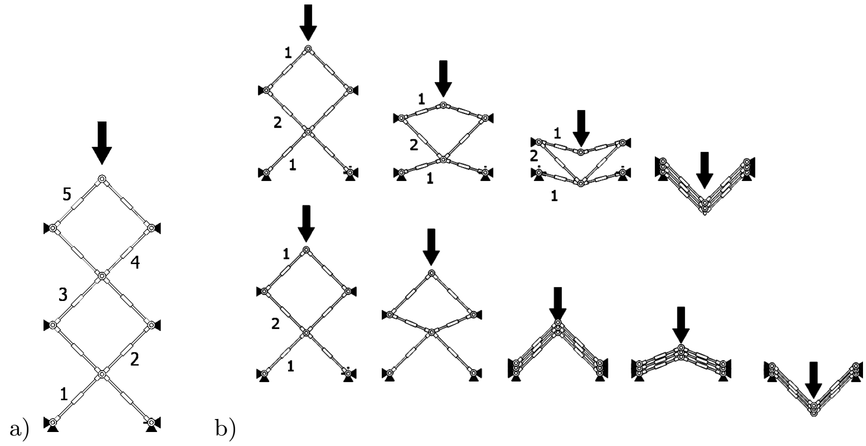

Fig. 11. a) Single column MFM "5" and b) "2" with sample folding sequences

Simplified Model

In order to increase numerical efficiency of the analysis, a simplified analytical model was introduced. The model describes behaviour of a single column of the multifolding microstructure and is based on the following assumptions:

- the material is considered as a rigid-perfectly plastic, therefore the force in a deforming structural member remains constant,
- the mass of the absorber is negligible comparing to the impacting mass,
- the process of deformation is divided into sequences: in each sequence only one level of the microstructure is folding while the other elements remain rigid.
- the dissipated energy related to the final deformation is equal to the initial value of the kinetic energy

Such assumptions allow us to formulate the equilibrium equation based on the second Newton's law for the loaded node, for each sequence:

$$F\left(q(t)\right) = m\ddot{q}\left(q(t)\right) \tag{28}$$

Because the force in the elements, which are active in the current sequence, is constant, the change in the resultant force is dependent only on the change in the geometry of the elements. For the current sequence, this yields:

$$F\left(q\right) = 2\sigma^{\star}_{i,\mathrm{act}} A \sin\theta\left(q\right) = 2\sigma^{\star}_{i,\mathrm{act}} A \frac{h-q}{\sqrt{h^2-\left(h-q\right)^2}}, \quad q \in \langle 0, 2h\rangle \tag{29}$$

Acceleration of the loaded node can be expressed as:

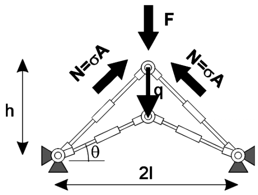

Fig. 12. Kinematics assumed for the simplified model

$$\ddot{q}\left(q\right)=\frac{F\left(q\right)}{m},\quad q\in\langle 0,2h\rangle \tag{30}$$

Energy of the plastic strain is equal to the work of the resultant force F

$$Dyss^{\mathrm{seq}}=E_{\mathrm{plast}}=\int\limits_{0}^{2h}F\left(q\right)dq \tag{31}$$

To describe the behaviour of the single-column structure one has to solve the following problem: *for a given set of design parameters* $\boldsymbol{\sigma}$ *and kinetic energy* E_{kin} *find corresponding evolution of acceleration of the loaded node and total energy of the plastic strain* $E_{plast}=\sum\limits_{1}^{Nseq}Dyss^{seq}$.

The initial distribution of the yield stresses $\boldsymbol{\sigma}$ uniquely defines evolution of the deformation. In each sequence elements with the lowest value of yield stress are folding first. The process evolves until the dissipated energy E_{plast} exceeds the initial kinetic energy E_{kin}.

The simplified model provides accuracy in predicting the acceleration and dissipation combined with a very good performance of the algorithm.

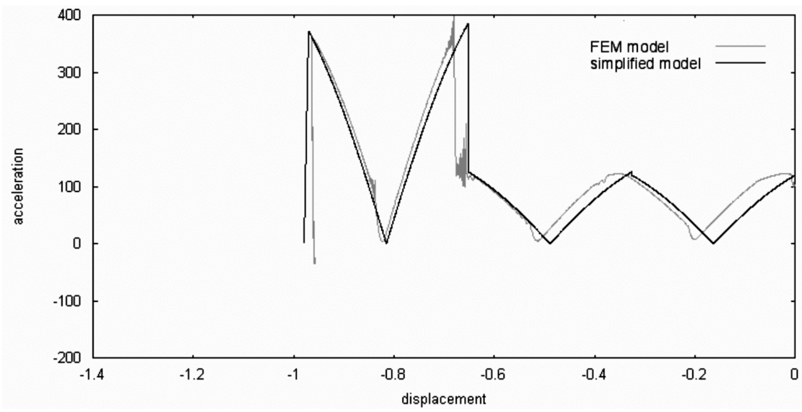

Fig. 13. Results from the simplified and FEM analysis

Control of the MFM

Process of structural adaptation to the impact consists of following stages:

- Load identification
- Selection of optimal strategy of dissipation
- Adaptation of active elements
- Dissipation of the energy

In order to provide optimal results of adaptive impact absorption one has to apply a correct control strategy. Two different approaches, discussed previously in [7], are considered: semi-active and active control. In the first case pre-selected triggering stress levels σ_i in structural elements remain unchanged during an impact, in the second one theoretical possibility of real-time changes in the control parameters is assumed.

- Semi-active control
  The objective function of the semi-active control is to minimize the maximal acceleration of the loaded node:$f = \min\left(\max_t \left\{\ddot{q}_{\mathrm{load}}(t)\right\}\right)$, with constraints imposed on control parameters $\sigma_i \in \langle\sigma_{\min}, \sigma_{\max}\rangle$ and maximal displacement $q_{\mathrm{load}} \leq q_{\max}$.
  Figure 14 presents results obtained for the basic absorber "2" (with and without applied control) compared with results for enhanced absorber "5". It is clearly visible that the multi-layered structure offers much better performance in a very wide range of kinetic energy value (different mass value with constant initial velocity of 15 m/s).

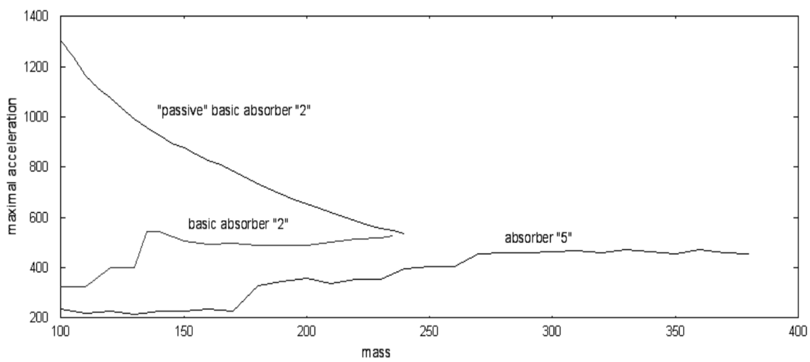

Fig. 14. Semi-active control results for absorbers "2" and "5" for different mass values

- Active control
  Objective function: for every time step, minimize the difference between acceleration in the loaded node and desired response function $\ddot{q}^d$: $\min f(t) = \left[\ddot{q}_{\mathrm{load}}(t) - \ddot{q}^d(t)\right]^2$.

Desired level of acceleration $\ddot{q}^d(t)$ must provide dissipation of kinetic energy of the impact. Therefore, it is necessary to solve an additional optimization task (in this case, based on a simplified approach).
In FEM analysis, at every time step control parameters (hardening coefficient in plasticity model) are updated on the basis of the objective function's gradient, which is calculated by the finite difference approach. A classical algorithm of the steepest gradient descent is applied.
Results, obtained for the basic absorber "2", are presented in Figs. 15 and 16. The strategy of active control is compared to the semi-active and passive one. Active control further improves the performance of the absorber by 20%-30%. Nevertheless, it has to be taken into consideration if such improvement can justify application of a complicated strategy in comparison with the simple and robust semi-active approach.

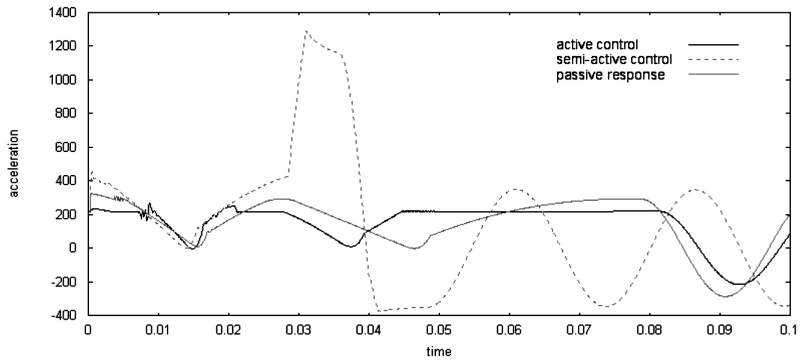

Fig. 15. Results of different control strategies for impacting mass of 100 kg

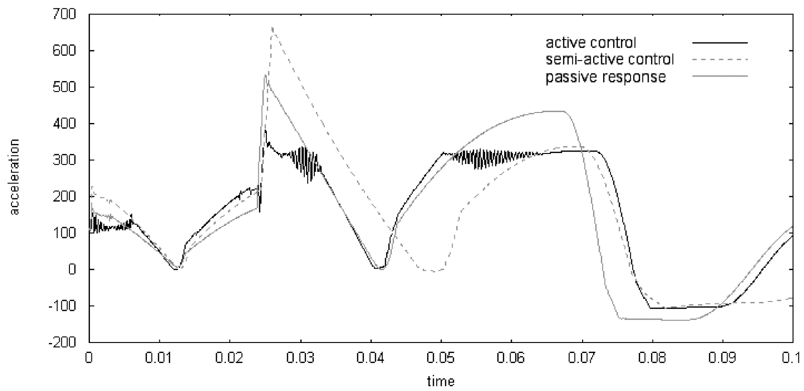

Fig. 16. Results of different control strategies for impacting mass of 200 kg

6 Conclusions

The methodology (based on the so-called Dynamic Virtual Distortion Method) of the design of structures exposed to impact loading is presented in the work. Minimization of material volume and accelerations of structural response are chosen as the objective functions for optimal design of structures adapting to impact loads. The cross-sections of structural members as well as stress levels triggering plastic-like behavior and initial prestressing are the design parameters.

The crucial points in the proposed methodology are:

- development of technique for detection and identification of the impact in advance,
- development of devices for quickly responding, controlled in real time, semi-active detachment of structural members and for introducing initial distortions,
- another challenging field for research is mentioned above development of methodology able to determine optimal control strategies for impact absorption in multi-load cases, when compromise solutions are required.

The paper demonstrates the effectiveness of the proposed concept. The yield stress level adaptation to the applied load has significant influence on the intensity of impact energy dissipation.

The concept of adaptive MFM systems has been discussed, where the following general methodology in the design can be proposed:

- design a topological pattern and material redistribution of the adaptive structure with given initial configuration for variety of all expected extreme loadings,
- particularly, consider the MFM pattern in case of frontal impacts,
- apply in real time the pre-computed control strategy as the response for detected (through a sensor system) impact.

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